Evaporating black holes and long range scaling

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Abstract

For an effective treatment of the evaporation process of a large black hole the problem concerning the role played by the fluctuations of the (vacuum) stress tensor close to the horizon is addressed. We present arguments which establish a principal relationship between the outwards fluctuations of the stress tensor close to the horizon and quantities describing the onset of the evaporation process. This suggest that the evaporation process may be described by a fluctuation-dissipation theorem relating the noise of the horizon to the black hole evaporation rate.

1 Introduction

One of the central questions in the theory of black hole evaporation concerns the detailed understanding of the characteristics scales involved in the treatment of the onset of evaporation process. In the usual treatment the characteristic scale of length for a black hole of mass M is identified with the Schwarzschild radius[2] $2M$ which for a large black hole is a macroscopic length. Therefore, for a large black hole one may be inclined to persist on the paradigm of an effective description requiring a low energy treatment of black hole evaporation involving only the characteristic energy scale $\sim \frac{1}{2M}$.

The point, however, is that due to the infinite gravitational red-shift on the horizon, the long time (and long distance) observations in the outside region of a black hole exhibit correlations with the physical situations in a high energy regime in the vicinity of the horizon where the fluctuations of the (vacuum) stress tensor due to the high energy

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[2]We use units in which $G = c = \hbar = 1$.

gravitational effects can no longer be neglected [1][2][3]. This remark demonstrates that a low energy description involving only the characteristic scale $\frac{1}{2M}$ may not encompass the essential features of black hole evaporation. In fact it seems that high energy gravitational effects may profoundly affects the correct form of the stress-tensor fluctuations near the horizon in such a way that applicability of semiclassical methods for the treatment of outside observations becomes questionable.

A promising idea towards clarifying this issue comes from the application of the black hole complementarity principle [4] [5], which emphasizes that the role played by the high energy effects near the horizon depends essentially on whether we look at physical states from the outside or the inside of the horizon. For example, in the Hilbert space used in the description of an outside static observer, the principle demands that the high energy gravitational effects decouple themselves in form of a materialized stretched horizon where the incoming information are transferred into the outgoing thermal radiation. Thus as long as physical states are strictly localized outside the (stretched) horizon one may assume that the high energy gravitational effects are suppressed in such a way that the correct form of the stress-tensor fluctuations near the horizon can be taken to posses only the appropriate low energy characteristics of a small disturbance of black hole causal structure, so that semiclassical methods may be applied to a good approximation. The conditions of this effective Hilbert space outside the horizon however can not be indicative of the observations made by an infalling observer crossing the horizon because the high energy effects never seem to decouple from the inside of the horizon so that the Hilbert space becomes the wrong Hilbert space for the fundamental low energy description of the physical state of an infalling observer. To avoid any physical inconsistency one requires that there is basically no way to combine the description of the outside observations with the description of observations made by infalling observers crossing the horizon, the two descriptions are complementary descriptions. In this way complementarity is claimed to reflect an important feature of black hole evaporation.

For a detailed understanding of the principle of black hole complementarity it is important to realize that the principle essentially implies two things. Firstly it implies an assertion about the impossibility of realizing the physical state of an outside static observer and the physical state of an infalling observer crossing the horizon in the same Hilbert space. In fact, the two states are related to basically different sets of boundary conditions inside the horizon. This is a statement about the complementary properties of basically different Hilbert spaces used by observers separated by mutually exclusive behavior of their coordinates on the horizon. The second implication is that for the effective description of observations outside the horizon the principle requires a systematic link between a small disturbance of black hole causal structure and the evaporation process through the choice of a physical low energy state. In order to establish such a link we shall study a model in which the stress-tensor fluctuations near the horizon, expressed in coordinates of an outside static observer, are taken to posses the appropriate low energy characteristics of random fluctuations arising from the noise of gravitational effects inside the horizon (the horizon noise). According to the principle of black hole complementarity this effective ansatz, which is studied in this paper, is correct as long as the physical state of an static observer is strictly localized outside the horizon. Inside the horizon the methods of this effective ansatz are generally felt to break down, because the correct form of the physical state of an static observer inside the horizon never seem to posses the low energy characteristics required by that effective ansatz.

This effective ansatz is very useful for posing a general question concerning the dissipative effects of quantized fields in the presence of a black hole. We would expect, namely, that the random fluctuations of the stress tensor near the horizon, expressed in coordinates of an outside static observer, to posses a dissipative character. This kind of behavior is suggested in a very general way by fluctuation-dissipation theorems which systematically link the random fluctuations of a system to a systematic effect, namely the dissipative behavior of the same system over long time intervals. In the present context the dissipation is represented by the black hole evaporation process. Therefore it is important to determine how the random fluctuations of the stress tensor near the horizon can be related by the conditions of a semiclassical theory to the evaporation rate. In this paper we shall study this relationship using a two-dimensional Schwarzschild black hole model. The significance of such a lower dimensional model lies in the fact that it may be considered as a model arising from the geometric optics approximation of a physical spherically symmetric black hole model. Such a restriction to geometric optics approximations and to the corresponding lower dimensional methods simplifies considerably the analysis, and it is generally believed that the qualitative features of the evaporation process will not alter too much by this restriction. The organization of the paper is as follows: In the subsequent two chapters we present the model and discuss heuristic arguments leading to a long range scaling law which controls the outwards stress-tensor fluctuations near the horizon, expressed in the coordinates of an outside static observer, in terms of a large correlations length. In this model we deal with the mean value of these fluctuations which is taken to be systematically determined by the outwards component of the renormalized expectation value of the stress tensor of a quantum field taken in some appropriately chosen quantum state. Therefore the scaling law should basically understood as a condition imposed on this state. In chapter 4 we present a dynamical derivation of the scaling law on the basis of the backreaction effect using a Planckian cutoff condition in the frame of an observer who uses finite coordinates at the horizon. In chapter 5 we show that the scaling law can be represented in form of a fluctuation-dissipation theorem which relates the mean value of the outwards fluctuations of the stress tensor near the horizon to the black hole evaporation rate. Some arguments for deriving corrections to the radiation temperature are then presented in chapter 6. The paper ends with some concluding remarks.

2 The Model

We consider a two-dimensional analog of the Schwarzschild black hole of mass M, described in coordinates which are indicative of the outside observations (Schwarzschild coordinates) by the metric

$$ds^2 = -\Omega(r)dt^2 + \Omega^{-1}(r)dr^2, \quad \Omega(r) = 1 - 2M/r, \tag{1}$$

to which a massless scalar quantum field ϕ is taken to be minimally coupled. Let the stress tensor of ϕ in the (t,r) coordinates be denoted by $T_{\mu\nu}$. We are primarily interested in the form of the low energy fluctuations of $T_{\mu\nu}$ near the horizon, i.e. in the effective horizon limit $r \to 2M$. In a semiclassical treatment these fluctuations must be considered as random fluctuations due to the horizon noise. Denoting their mean value by $\delta T_{\mu\nu}$, the first fundamental task is how to control the typical value of $\delta T_{\mu\nu}(r \to 2M)$ in terms of quantities accessible to a semiclassical treatment.

In a semicalssical theory the operator $T_{\mu\nu}$ arises as a singular operator because it involves the product of the field operator at a single point. Therefore one can generally assume that the typical value of $\delta T_{\mu\nu}(r \to 2M)$ may be related to the effective horizon limit $r \to 2M$ of the renormalized expectation value $< T_{\mu\nu} >^{ren.}_{\omega}$ taken in some appropriately chosen quantum state ω, namely

$$\delta T_{\mu\nu}(r \to 2M) \sim < T_{\mu\nu}(r \to 2M) >^{ren.}_{\omega} . \tag{2}$$

This relation links two kinds of effects. The right hand side of (2) is the systematic value of the stress tensor near the horizon which is accessible to a semiclassical treatment through the choice of the quantum state ω, whereas the left hand side is the random value due to random fluctuations. The relation (2) requires a systematic link between both values. The quantum state ω is, therefore, assumed to link the random value of the stress tensor $T_{\mu\nu}$ near the horizon with its systematic counterpart, namely the renormalized expectation value of $< T_{\mu\nu}(r \to 2M) >^{ren.}_{\omega}$. This has an essential consequence for the characterization of the state ω.

3 Long range scaling

For the characterization of the state ω the determination of the outwards component of $< T_{\mu\nu} >^{ren.}_{\omega}$ near the horizon is very important because this component is the indicative quantity of the long-time observations in the outside region. Let $< T_{uu} >^{ren.}_{\omega}$ denotes the outwards component of $< T_{\mu\nu} >^{ren.}_{\omega}$ defined with respect to the standard outward (retarded) time u of the metric (1), namely

$$u = t - \overset{*}{r}, \qquad \overset{*}{r} = r + 2M \ln |\frac{r}{2M} - 1|. \tag{3}$$

The problem is how to determine the limit $< T_{uu}(r \to 2M) >^{ren.}_{\omega}$. The relation (2) tells us that $< T_{uu}(r \to 2M) >^{ren.}_{\omega}$ is related to $\delta T_{uu}(r \to 2M)$ which is the mean value of the outwards fluctuations of the stress tensor near the horizon. There is an argument, based on the black hole causal structure, which suggests that these fluctuations are correlated over all scales of lengths. The argument goes as follows: The black hole causal structure implies that null rays which are equispaced along the future null infinity over long distances crowd up near the horizon over small distances. This requires that the outwards fluctuations of the stress tensor near the horizon shall be correlated over almost all scales of lengths. In particular the typical scale of the outward component of the renormalized stress tensor near the horizon may be taken to be set by a large correlation length ξ which could have in principle its value many orders of magnitude away from the Schwarzschid radius, giving us the the scale hierarchy

$$\xi >> 2M. \tag{4}$$

The basic strategy is now to express the limit $< T_{uu}(r \to 2M) >^{ren.}_{\omega}$ in terms of the correlation length ξ using dimensional arguments. In doing this we should take into account that, due to the scale hierarchy (4), any physical quantity may depend in principle on the dimensionless ratio $2M/\xi$. For the limit $< T_{uu}(r \to 2M) >^{ren.}_{\omega}$ we get therefore on dimensional grounds the general relation

$$< T_{uu}(r \to 2M) >^{ren.}_{\omega} = \xi^{-2} f(2M/\xi) \tag{5}$$

where f is a scaling function. The particular choice of the scaling function f depends on the state ω and the corresponding Hilbert space. For physically admissible states however we expect that the outwards component of the renormalized stress tensor near the horizon does not exhibit a significant sensitivity to a change of the dimensionless scaling variable $2M/\xi$ as long as this variable remains small according to (4). This means that the scaling function may be approximated by a constant function. In this way the geometric length $2M$ in the relation (5) drops out so that the significant scale is taken to be the correlation length ξ only. We are therefore led to predict the long range scaling law

$$< T_{uu}(r \to 2M) >^{ren.}_{\omega} \sim \xi^{-2}. \tag{6}$$

We should emphasize that the scaling law (6) as it stands is predicted on the basis of heuristic arguments, and a systematic framework for its justification is still missing. Regarding this point the following remark is necessary. One may study the scaling law (6) from the viewpoint of the renormalization group arguments. For this purpose the outwards fluctuations of the stress tensor expressed in the coordinates of a static observer near the horizon should properly be taken to be correlated over a static cutoff length near the horizon, i.e. a cutoff length used by a static observer properly located just outside the horizon. In this way the correlation length ξ appears to be systematically linked with the value of such a static cutoff[3]. The requirement that an observable quantity such as the left hand side of (6) should be cutoff-independent appears then to be in conflict with the scaling law (6). In the present case the inconsistency of (6) with the standard renormalization group arguments does not reflect a weakness of our argumentations. In fact the standard renormalization group arguments are not applicable in the present case where, due to the black hole causal structure, a strong sensitivity of the outwards fluctuations near the horizon to the cutoff mechanism can be expected. This point is of particular importance for an understanding of the scaling law (6). It indicates that a systematic framework for the justification of the scaling law (6) can not be based on the standard renormalization group arguments, so another methods must be applied. We shall deal with this issue in the next section where a dynamical derivation of (6) is given. This derivation is mainly based on two assumptions, one involving a Planckian cutoff condition in the frame of an observer who uses finite coordinates at the horizon and one involving the backreaction effect. Although this derivation may not seem to be conclusive, but it emphasizes the fact that the scaling law (6) may systematically be linked with the

[3]Actually a static cutoff length near the horizon is significantly large because a static cutoff frequency for outgoing modes tends to zero at the horizon [2]

physical mechanism of a cutoff. In the remaining part of this section we collect some theoretical facts in connection with (6).

It is important to note that the scaling law (6) implies that the characteristic order of the magnitude of the expectation value $< T_{uu}(r \rightarrow 2M) >_{\omega}^{ren.}$ is set by the correlation length of the outwards fluctuations of the stress tensor near the horizon, which is distinctly separated by the scale hierarchy (4) from the characteristic macroscopic length of the system, namely $2M$. This feature implies that the quantity $< T_{uu}(r \rightarrow 2M) >_{\omega}^{ren.}$ should basically decouple from the dynamical constraint of the renormalization theory describing the effective change of the late-time configuration of the renormalized expectation value $< T_{\mu\nu} >_{\omega}^{ren.}$, because this change can generically be expected to occur on those typical macroscopic length scales which are distinctly much smaller than ξ. That this decoupling actually happens to be the case may be seen from the following consideration:
The dynamical constraint of the renormalization theory can be expressed in form of a hydrodynamic constraint, namely the conservation law

$$\nabla^{\mu} < T_{\mu\nu} >_{\omega}^{ren.} = 0. \tag{7}$$

This law can be used to determine the static form of $< T_{\mu\nu} >_{\omega}^{ren.}$. In doing this we ignore effects related to a preassigned time-dependence of the expectation value $< T_{\mu\nu} >_{\omega}^{ren.}$. Naturally, we assume that any time-dependence of $< T_{\mu\nu} >_{\omega}^{ren.}$ should be suppressed in the late-time limit. Therefore we look for the static configuration of the expectation value $< T_{\mu\nu} >_{\omega}^{ren.}$ which can be found to be [6][7]

$$< T_{\mu}^{\nu} >_{\omega}^{ren.} = T_{\mu}^{(1)\nu} + T_{\mu}^{(2)\nu} + T_{\mu}^{(3)\nu} \tag{8}$$

where in $(t, \overset{*}{r})$ coordinates

$$T_{\mu}^{(1)\nu} = \begin{pmatrix} < T_{\alpha}^{\alpha}(r) >_{\omega} - \Omega^{-1}(r) H(r) & 0 \\ 0 & \Omega^{-1}(r) H(r) \end{pmatrix} \tag{9}$$

$$T_{\mu}^{(2)\nu} = \Omega^{-1}(r) \frac{K}{M^2} \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix} \tag{10}$$

$$T_{\mu}^{(3)\nu} = \Omega^{-1}(r) \frac{Q}{M^2} \begin{pmatrix} -2 & 0 \\ 0 & 2 \end{pmatrix}. \tag{11}$$

Here K and Q are arbitrary constants, and

$$H(r) = \frac{1}{2} \int_{2M}^{r} \left(\frac{d}{dr'} \Omega(r') \right) < T_{\alpha}^{\alpha}(r') >_{\omega} dr'. \tag{12}$$

We can now determine the outwards component of this solution. We find

$$< T_{uu} >_{\omega}^{ren.} = \frac{1}{2}(H(r) + 2Q/M^2) - \frac{1}{4}\Omega(r) < T_{\alpha}^{\alpha}(r) >_{\omega} \tag{13}$$

which near the horizon yields the scaling law

$$< T_{uu}(r \rightarrow 2M) >_{\omega}^{ren.} \rightarrow Q/M^2. \tag{14}$$

Thus, if the conservation law is applied, it is possible to describe the outwards component of $< T_{\mu\nu} >_{\omega}^{ren.}$ near the horizon in terms of an integration constant which is not dependent upon the particular dynamical coupling of ϕ to the metric (1). This implies that the specification of that component must reflect a model independent general characteristic of the vacuum state as observed by those observers which are strictly localized outside the black hole. This observation establishes the decoupling of $< T_{uu}(r \to 2M) >_{\omega}^{ren.}$ from the dynamical constraint of the renormalization theory.

4 The dynamical derivation of the scaling law

The heuristic arguments that led to the scaling law (6) can find a dynamical justification by combining a cutoff condition near the horizon with the backreaction effect of black hole thermal radiation. To this aim we first note that the unrenormalized expectation value $< T_{uu}(r \to 2M) >_{\omega}$ is mathematically a singular quantity. The corresponding renormalized value can be obtained firstly by introducing a Planckian cutoff length $l_c \sim 1$, and secondly by specifying the reference frame to which the cutoff is applied. The most natural reference frame for the imposition of a Planckian cutoff is a reference frame of an infalling observer crossing the horizon. The coordinate system which is indicative of such an observer is a coordinate system which is finite at the horizon, such as the inwards and the outwards Kruskal time-coordinates, defined respectively as

$$V = 4M\ exp(v/4M), \quad U = -4M\ exp(-u/4M) \tag{15}$$

where $v = t + \overset{*}{r}$ is the advanced time and u is the retarded time as given by (3). In this coordinate system the imposition of a Planckian cutoff on the expectation value $< T_{uu}(r \to 2M) >_{\omega}$ is taken to correspond to the requirement that the stress-tensor fluctuations near the horizon shall be correlated over a cutoff length of Planckian size. The correlation length of these fluctuations may therefore be taken to be $l_c \sim 1$. This length sets the typical scale of length for the determination of $< T_{UU}(r \to 2M) >_{\omega}^{ren.}$. Therefore on dimensional grounds we arrive at the relation

$$< T_{UU}(r \to 2M) >_{\omega}^{ren.} \sim (l_c)^{-2} \sim 1 \tag{16}$$

which indicates that the state ω exhibits large stress-tensor fluctuations in the frame of an infalling observer crossing the horizon. It is important to note that this feature which arises from the cutoff condition reflects the characteristic feature of the black hole complementarity principle because it indicates that strong gravitational effects may not decouple from the inside of the horizon, so that the Hilbert space of the state ω becomes the wrong Hilbert space for the low energy description of the physical state of an infalling observer.

To derive the scaling law (6) from the relation (16) we first relate the expectation value $< T_{UU} >_{\omega}^{ren.}$ to $< T_{uu} >_{\omega}^{ren.}$ using the coordinate transformation (15) to find

$$< T_{UU} >_{\omega}^{ren.} = \frac{1}{4}\ exp(-r/M) V^2 (r - 2M)^{-2} < T_{uu} >_{\omega}^{ren.} . \tag{17}$$

We then try to use this relation for the derivation of the scaling behavior of $< T_{uu} >_{\omega}^{ren.}$ in the limit $r \rightarrow 2M$. The first observation is that (17) together with (16) implies that $< T_{uu} >_{\omega}^{ren.}$ vanishes in the limit $r \rightarrow 2M$. However this feature is an idealization of neglecting the backreaction due the Hawking effect. The consideration of the backreaction implies that the effective horizon limit $r \rightarrow 2M$ should be carried out with respect to a mass scale which is slightly smaller than the mass M, namely

$$r \rightarrow 2(M - \delta M) \tag{18}$$

where δM is of the order of the mass evaporated away during the times just prior to the formation of the horizon. For a sufficiently large black hole one can generally expect that

$$\delta M << M \tag{19}$$

holds. But it can be shown that δM is even much smaller than the Planck mass, namely $\delta M << 1$. To show this we consider M as a function of the advanced time $M(v)$, and let v_0 be the value of the advanced time at which the horizon would form if we neglect the backreaction effect. The mass δM can be estimated by

$$\delta M \sim |\frac{dM}{dv}(\tilde{v})|(v_0 - \tilde{v}) \tag{20}$$

where the time $\tilde{v}$ is taken to characterize the onset of the evaporation process, so it must be very close to the horizon formation time v_0. In general the time difference $\delta v = v_0 - \tilde{v}$ must be taken as much smaller than the characteristic time-scale of the system which is set by the black hole mass. Therefore one should have

$$\delta v << 2M. \tag{21}$$

Although the validity of this relation seems to be apparent from the context, but it can be justified also by noting that, for a value $\tilde{v}$ of the advanced time characterized by (21), a null-geodesic after its propagation through a collapsing objects will be characterized by a retarded time $\tilde{u} \sim -4M \ln{(v_0 - \tilde{v})}/2M$; which is characteristic to the onset of the evaporation process [8][3]. The correct order of magnitude of δM can now be determined if we estimate $\frac{dM}{dv}(\tilde{v})$ by the Hawking law

$$-\frac{dM}{dv} \sim \frac{1}{M^2}. \tag{22}$$

Using this law in (20) we obtain

$$\delta M \sim \frac{1}{M}\frac{\delta v}{M} \tag{23}$$

which in conjunction with (21) yields

$$\delta M << 1. \tag{24}$$

This relation can be used to estimate the renormalized expectation value $< T_{uu} >_{\omega}^{ren.}$ near the horizon using the effective horizon limit $r \rightarrow 2M - \delta M$ of the relation (17) together with (16). We obtain

$$< T_{uu}(r \rightarrow 2M) >_{\omega}^{ren} \sim \xi^{-2} \tag{25}$$

where

$$\xi \sim \frac{2M}{\delta M} >> 2M. \tag{26}$$

We arrive therefore at the scaling law (6).

5 A fluctuation-dissipation theorem

The scaling law (6) links the scaling behavior of the renormalized expectation value $< T_{uu} >_{\omega}^{ren.}$ near the horizon with the large correlations length ξ of the stress-tensor fluctuations near the horizon through the choice of the quantum state ω in the outside region of the black hole. One can therefore expect this state ω to posses a dissipative character related to these fluctuations. To see this we first write (6) in the form

$$< T_{uu}(r \to 2M) >_{\omega}^{ren.} \sim \gamma M^{-2} \tag{27}$$

where $\gamma \sim (2M/\xi)^2 << 1$. Now combining (27) with the the Hawking law (22) we get

$$< T_{uu}(r \to 2M) >_{\omega}^{ren.} \sim -\gamma \frac{dM}{dv}. \tag{28}$$

This formulation of the scaling law (6) is instructive because it shows that the quantum state ω links the outwards fluctuations of the stress tensor near the horizon to a long-time dissipative behavior, namely the evaporation rate of the black hole. Therefore, the scaling law if combined with the Hawking law may be brought into a form suggesting a fluctuation-dissipation theorem. One can alternatively consider the arguments presented in the previous chapter as demonstrating as to how such a theorem can be derived on dynamical basis from a cutoff condition near the horizon.

It is important to add the following remarks concerning the correct physical interpretation of (28). The fluctuations that contribute to the left hand side of (28) cause a small disturbance of the black hole metric and the corresponding causal structure. The relation (28) describes how this disturbance is related to the evaporation rate via the choice of the quantum state ω for the undisturbed system, i.e, the black hole thermal state. This is is very much in the spirit of the general framework of the response theory which relates the response of a system to a small disturbance to the equilibrium characteristics of the undisturbed system.

6 Corrections to the Hawking effect

It may be of interest to examine the effect of the scale-separation $\xi >> 2M$ on the Hawking effect. Generally, one expects to find a deviation of the black hole temperature from the Hawking temperature by a term of the relative order $(2M/\xi)^{\alpha}$ where α is a characteristic exponent. To determine this exponent we proceed as follows: In two dimensions an outwards flux of thermal radiation can be characterized at large r by the energy momentum tensor

$$\frac{\pi}{12} T^2 \begin{pmatrix} -1 & -1 \\ 1 & 1 \end{pmatrix} \tag{29}$$

in which T is the temperature. From (29) one infers that a static, spherically symmetric configuration of matter which describes the Hawking radiation at large r must have a stress tensor satisfying the condition

$$T_t^t(r) \to T_t^r(r), \quad as\ r \to \infty \tag{30}$$

which means that the energy density and the flux are asymptotically equal. If this condition is applied to the general solution (8) one gets

$$K - \frac{1}{2}M^2[H(\infty) - < T_\alpha^\alpha(\infty) >_\omega + 2Q] = 0. \tag{31}$$

To derive the Hawking radiation from such a relation one usually assumes two additional requirements. The first one corresponds to the consistency of the trace anomaly with respect to the two dimensional metric (1), namely [6]

$$< T_\alpha^\alpha(r) >_\omega^{ren.} = \frac{M}{6\pi r^3}. \tag{32}$$

The second requirement concerns the finiteness of the energy momentum tensor at the horizon with respect to a coordinate system which is finite there. In order to implement the second assumption the standard derivation takes the value $Q = 0$ which arises as a pure effect of the transformation law (17) in the limit $r \to 2M$. There is however some objections for considering the value $Q = 0$ as the correct one, because the finiteness condition of a quantum stress tensor at the horizon requires us to investigate a cutoff condition in the frame of an observer who uses finite coordinates at the horizon, and we have seen that this leads to the scaling law (6) which together with (14) predicts a non-vanishing value $\sim (2M/\xi)^2$ for Q. Thus for the derivation of the Hawking radiation we may take the consistency of the trace anomaly together with this value of Q. At large r the latter condition predicts via the last term in (8), namely the tensor $T_\mu^{(3)\nu}$, a deviation of the thermal radiation from the Hawking temperature of the relative order $(2M/\xi)^2$, leading to the characteristic exponent $\alpha = 2$ which coincides in the present case with the dimensionality of space-time.

We should also remark that the non-vanishing value $Q \sim (2M/\xi)^2$ predicts that the expectation value $< T_{\mu\nu} >_\omega^{ren.}$ has at large r a term corresponding to a background heat bath with the temperature $\sim 1/\xi$. This follows if one compares the tensor $T_\mu^{(3)\nu}$ at large r with the stress tensor of an equilibrium gas, namely

$$\frac{\pi}{12}(kT)^2 \begin{pmatrix} -2 & 0 \\ 0 & 2 \end{pmatrix}. \tag{33}$$

Such a model may have some power at the cosmological level, in that the background heat bath may act as a model for the thermal equilibrium gas of an associated cosmological horizon. In this way the cutoff condition on the horizon may be linked with a small cosmological constant.

7 Concluding remarks

The paper has examined a new method for introducing a quantum state ω for the outside region of a black hole. The distinct feature of this method as compared with the standard methods for introducing black hole states, such as those discussed in [6] [7], is that it links via the scaling law (6) the choice of the quantum state ω outside the horizon with the noise of gravitational effects inside the horizon, and in this respect it emphasizes a general relationship between a small disturbance of the black hole causal structure and the choice of an external quantum state in the absence of this disturbance. One may expect that the implications that arise from this viewpoint may improve our uderstanding about the nature of black hole evaporation.

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